Influence of Support Oscillation in Dynamic Stability Tests

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A tractable but quite general analysis of the influence of support oscillation in oscillatory wind tunnel testing is presented. This is an extension of a recent analysis of sting plunging in the pitch-oscillation mode. It is shown that the sting plunging correction is equivalent to an aerodynamic axis transformation from the inertial rotation center. The basic approach is free of simplifying assumptions beyond those implicit in the transformation equations and can be extended to any measurement degree of freedom. The only requirement is for appropriate measurements of the location of the axis of rotation.

Nomenclature

b = wing span
$\bar{c}$ = mean aerodynamic chord
$C_{i\bar{\omega}}$ = dynamic derivative with respect to reduced-rate parameter:
$$\frac{\partial C_i}{\partial \bar{\omega}}, \quad i=\ell,m,n; \; \omega=q, \; \dot{\alpha} \; (d=\bar{c}); \; \omega=r \; (d=b)$$
$C_{i\sigma}$ = static derivative with respect to σ:
$$\frac{\partial c_i}{\partial \sigma}, \quad i=\ell,m,n; \; \sigma=\alpha,\beta$$
C_ℓ = rolling-moment coefficient, $L/(q_\infty Sb)$
C_m = pitching-moment coefficient, $M/(q_\infty S\bar{c})$
C_n = yawing moment coefficient, $N/(q_\infty Sb)$
d = generalized reference length
F = generalized aerodynamic force vector
G = generalized aerodynamic moment vector
L,M,N = rolling, pitching, and yawing moments
p,q,r = body axes angular velocities
q_∞ = freestream dynamic pressure
Q = generalized pitching angular velocity
S = reference area
V_∞ = freestream velocity
x,y,z = body axes system, fixed at center of rotation
$\bar{X},\bar{Y},\bar{Z}$ = wind-fixed coordinate system
$\hat{x}$ = longitudinal body coordinate, positive aft
$\hat{x}_c$ = displacement of oscillation axis
Y,Z = aerodynamic side force and normal force
α,β = angles of attack and sideslip
$\bar{\alpha}$ = mean angle of attack
Δ = amplitude
θ = pitch perturbation angle
μ = $m/(\rho_\infty Sd)$
ζ,ξ = $|\hat{z}|/d$, $|\hat{x}|/d$
σ = generalized angle of incidence
Φ_z = phase angle between translational and rotational motions
ω = generalized angular velocity
$\bar{\omega}$ = reduced frequency, $=\omega d/(2V_\infty)$
$\bar{\omega}_{z0}$ = natural frequency in pure translation

Subscripts

c = center of rotation
cg = center of mass
f = flexure
y,z = in the presence of translation along y,z

Superscripts

$(\dot{\;})$ = partial differential with respect to time
$(\bar{\;})$ = composite fixed-axis derivative
$(')$ = referred to rotation center
$(^*)$ = transformed to new axis location

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Introduction

PREVIOUS efforts at approaching the sting plunging problem have involved solutions to the coupled two-degree-of-freedom (DOF) equations of motion.[1-3] Subsequently, it was shown that the equations can be simplified with the introduction of the location of the axis of rotation in an inertial frame as a parameter.[4]

In this paper, it is shown that the equations for the corrections to the measured derivatives due to sting plunging can be derived from an aerodynamic transformation between the inertial axis of rotation and the reference center and that the approach is not restricted to a particular measurement degree of freedom. Also discussed are experimental alternatives to the analytical correction procedure.

Background

Consider the planar oscillatory motion of a sting-model system as shown in Fig. 1. The choice of the pitch plane is convenient for comparison with previously published work but otherwise quite incidental, since, as will be shown herein, the analysis is applicable to 2-DOF oscillation in any plane of motion. The fact that the balance is translating in a forced-oscillation-in-pitch experiment is not accounted for in the data reduction, which implicitly assumes a single-DOF or fixed-axis rotation. Thus, corrections are required when the dynamic moments are resolved about a translating point.

The simplified solution to the coupled pitch-plunge equations of motion derived by Ericsson[1] for low-lift, noncritical sting stiffness and low structural damping yields the following derivatives in the presence of the plunging DOF:

$$\bar{C}_{mq\dot{z}} = \bar{C}_{mq} - 2\,\frac{\Delta\zeta}{\Delta\theta}\,C_{m\alpha}\cos\Phi_z \tag{1}$$

$$C_{m\theta z} = C_{m\alpha}\left[1-\bar{\omega}^2\left(\frac{\Delta\zeta}{\Delta\theta}\right)^2\right], \quad \frac{\Delta\zeta}{\Delta\theta} = \frac{C_{L\alpha}}{\mu}\Big/|\bar{\omega}_{z0}^2-\bar{\omega}^2|$$

$$\Phi_z \simeq \cos^{-1}\left\{\frac{\bar{\omega}_{z0}^2-\bar{\omega}^2}{|\bar{\omega}_{z0}^2-\bar{\omega}^2|}\right\} \begin{matrix} =0; \; \bar{\omega}_{z0}>\bar{\omega} \\ =\pi; \; \bar{\omega}_{z0}<\bar{\omega} \end{matrix}$$

(The presence of the factor 2 in the second term arises from the definition of the reduced frequency $\bar{\omega}=\omega\bar{c}/(2V_\infty)$ used in the nondimensional form of the dynamic derivative. This

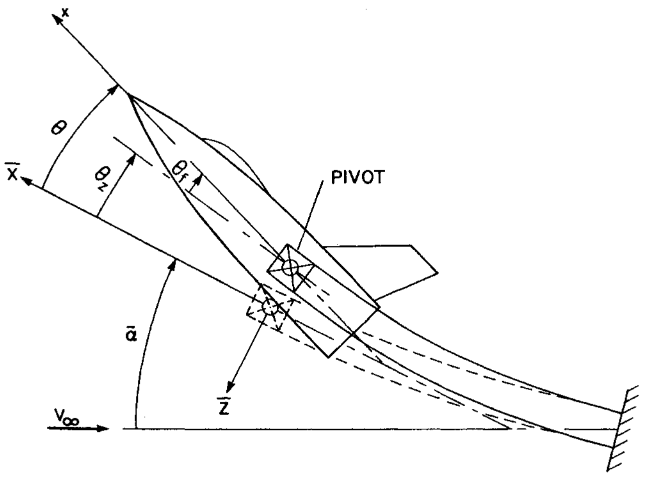

Fig. 1 Planar motion of sting-model system.

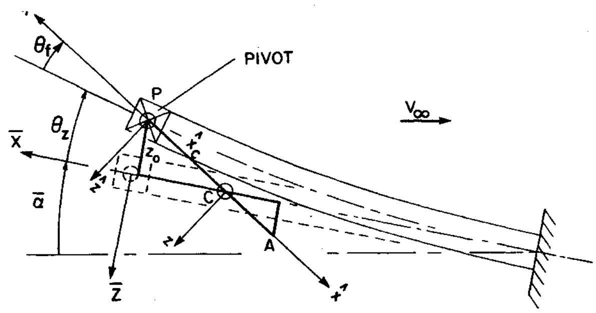

Fig. 2 Effective axis of oscillation.

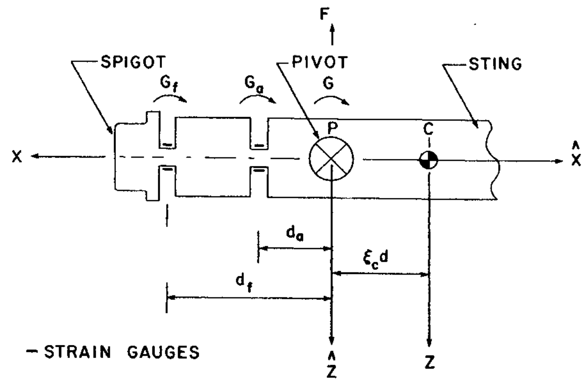

Fig. 3 Typical balance configuration.

differs from Ref. 4, where it was more convenient to use the form $\bar{\omega} = \omega \bar{c}/V_\infty$.)

It was shown by Beyers[4] that the single-DOF solution of Burt and Uselton[2] reduces to the same result at identical levels of approximation. Subsequently, the location of the axis of oscillation in a pitch/plunge motion $\hat{x}_c$ was derived from the sting-model geometry depicted in Fig. 2. The shift of the axis of rotation due to the translational DOF was shown to be

$$\xi_c = \frac{\Delta\zeta}{\Delta\theta} \tag{2}$$

Note that the axis displacement is defined to be positive aft while ξ_c is a positive quantity. As is naturally the case in small-amplitude oscillatory tests where θ_f is small (Fig. 2), ξ_c is assumed to be small.

When substituted in Eq. (1), the following simplified form was obtained:

$$\bar{C}_{mq\dot{z}} = \bar{C}_{mq} \pm 2\xi_c C_{m\alpha}, \quad \begin{cases} \bar{\omega}_{z0} < \bar{\omega} \\ \bar{\omega}_{z0} > \bar{\omega} \end{cases}$$

$$C_{m\alpha z} = C_{m\alpha}\{1 - (\xi_c \bar{\omega})^2\} \tag{3}$$

For small ξ_c and $\bar{\omega}$, the frequency term is negligibly small, $0(10^{-5})$, so that the set of equations becomes

$$\bar{C}_{mq\dot{z}} = \bar{C}_{mq} \pm 2\xi_c C_{m\alpha} \quad \begin{cases} \bar{\omega}_{z0} < \bar{\omega} \\ \bar{\omega}_{z0} > \bar{\omega} \end{cases}$$

$$C_{m\alpha z} \simeq C_{m\alpha} \tag{4}$$

In practical situations, the deficient static stability derivative due to sting plunging[1,2] usually results from the presence of an unbalanced mass and may be expressed as follows:

$$\bar{C}_{mq\dot{z}} = \bar{C}_{mq} \pm 2\xi_c C_{m\alpha} \quad \begin{cases} \bar{\omega}_{z0} < \bar{\omega} \\ \bar{\omega}_{z0} > \bar{\omega} \end{cases}$$

$$C_{m\alpha z} = C_{m\alpha} \pm 2\mu\xi_{cg}\xi_c(\bar{\omega}_{\text{off}}^2 - \bar{\omega}_{\text{on}}^2) \tag{5}$$

Analysis

From the discussion of the previous section, it is clear that a correction is required when the derivatives are resolved about a translating pivot P. On the other hand, when referred to the true rotation center C fixed in inertial space, the contribution of translational acceleration vanishes and no correction is required.

While the discussion is applicable to any balance configuration, the arrangement in Fig. 3, which is typical of moment-sensing balances commonly used in dynamic stability tests, is considered by way of illustration. The generalized aerodynamic force and moment vectors extracted from the alternating component of the balance signals are

$$F = \frac{G_a - G_f}{d_f - d_a}$$

$$G = \frac{G_a d_f - G_f d_a}{d_f - d_a} \tag{6}$$

referred to the reference axis at the pivot P. When resolved about the true rotation center, the measured moment is

$$G' = G + F x_c \tag{7}$$

Considering, for the moment, the pitch plane, then

$$M' = M - Z x_c \tag{8}$$

The static and dynamic derivatives are obtained from the real and imaginary parts of the moment:

$$\frac{\text{Im}(M')}{\omega\Delta\theta} = \frac{\text{Im}(M)}{\omega\Delta\theta} - \frac{\text{Im}(Z)}{\omega\Delta\theta} x_c$$

$$\frac{\text{Re}(M')}{\Delta\theta} = \frac{\text{Re}(M)}{\Delta\theta} - \frac{\text{Re}(Z)}{\Delta\theta} x_c \tag{9}$$

In coefficient form, Eqs. (9) yield

$$\bar{C}'_{mq} = \bar{C}_{mq\dot{z}} - \bar{C}_{Zq}\,\frac{x_c}{\bar{c}}$$

$$C'_{m\alpha} = C_{m\alpha z} - C_{Z\alpha}\,\frac{x_c}{\bar{c}} \tag{10}$$

These derivatives may be transformed to the reference center, which is nominally located at P. Using standard relations,[5] the transformation to a parallel coordinate system at a distance x_c from C yields (for zero sideslip)

$$\bar{C}_{mq} = \bar{C}'_{mq} + \frac{x_c}{\bar{c}}\,\bar{C}_{Zq} + 2\,\frac{x_c}{\bar{c}}\,\cos\alpha\left\{C'_{m\alpha} + \frac{x_c}{\bar{c}}\,C_{Z\alpha}\right\}$$

$$+4\,\frac{x_c}{\bar{c}}\,\sin\alpha\left\{C'_m + \frac{x_c}{\bar{c}}\,C_Z\right\}$$

$$C_{m\alpha} = C'_{m\alpha} + \frac{x_c}{\bar{c}}\,C_{Z\alpha}$$

$$C_m = C'_m + \frac{x_c}{\bar{c}}\,C_Z \tag{11}$$

Substituting Eqs. (10) in Eqs. (11),

$$\bar{C}_{mq} = \bar{C}_{mq\dot{z}} + 2\,\frac{x_c}{\bar{c}}\,\cos\alpha\; C_{m\alpha z} + 4\,\frac{x_c}{\bar{c}}\,\sin\alpha\; C_m$$

$$C_{m\alpha} = C_{m\alpha z} \tag{12}$$

Rearranging, the following equations are obtained for the effect of the translatory DOF on the pitch derivatives:

$$\bar{C}_{mq\dot{z}} = \bar{C}_{mq} - 2\,\frac{x_c}{\bar{c}}\,\cos\alpha\; C_{m\alpha} - 4\,\frac{x_c}{\bar{c}}\,\sin\alpha\; C_m$$

$$C_{m\alpha z} = C_{m\alpha} \tag{13}$$

Expressed in terms of ξ_c, Eqs. (13) yield

$$\bar{C}_{mq\dot{z}} = \bar{C}_{mq} \mp 2\xi_c\,\cos\alpha\; C_{m\alpha} \mp 4\xi_c\,\sin\alpha\; C_m$$

$$C_{m\alpha z} = C_{m\alpha} \tag{14}$$

where the negative and positive signs correspond to aft and forward displacements of the rotation center, respectively. If the system is rigidly mounted, the structural damping low and sting frequency noncritical, $\bar{\omega}_{z0} > 1.6\bar{\omega}$ or $\bar{\omega}_{z0} < 0.6\bar{\omega}$, the phase angle between the translational and rotational motions Φ_z will be 0 or π, corresponding to aft and forward shifts in the location of the axis of rotation, respectively. Hence, for small α, the effects of sting plunging are give by

$$\bar{C}_{mq\dot{z}} = \bar{C}_{mq} \pm 2\xi_c C_{m\alpha} \quad \begin{cases} \bar{\omega}_{z0} < \bar{\omega} \\ \bar{\omega}_{z0} > \bar{\omega} \end{cases}$$

$$C_{m\alpha z} \simeq C_{m\alpha} \tag{15}$$

which is precisely the result obtained in Eq. (4) solving the linear differential equations of motion with the assumptions of low lift and low reduced frequencies.

Therefore, the correction for the effect of sting plunging in the determination of direct pitching derivatives may be derived quite simply by transferring the measured moments to the rotation center, followed by an aerodynamic axis transformation to the reference center. The transformation equations, Eqs. (11), do not predict the frequency dependence of Eq. (3). At high frequencies or rates, a more complete model for the transformation of these derivatives to another center is required. Equation (3) could, of course, be used if necessary; however, such expressions can be avoided if the model/balance system is suitably balanced.

If valid transformation equations are available, the resulting corrections for sting plunging could be more complete than those derived from the equations of motion when the latter involve additional simplifying assumptions. Thus, Eqs. (14), e.g., although still based on linear aerodynamics, are more complete at finite α than the equations [(1) or (3)] used previously.

The present approach does not introduce any additional assumptions and is, therefore, applicable to any generalized aerodynamic parameters (including derivatives) and coordinate systems for which reliable transformations can be derived. Also, since the method does not require any knowledge of the motion of the support, it is applicable to all noncritical sting and strut configurations, provided that an absolute measurement of the axis of rotation can be made.

Aerodynamic Transfer Equations

In the case of pure rotational or translational motions,[6] exact transfer equations can be derived. However, this paper is primarily concerned with the correct data reduction of more conventional forced- or free-oscillation tests involving fixed-axis rotations. In this case, the dynamic derivative is the result of three component motions, namely, pure rotation, rate of change of α and β due to rotation, and translational acceleration. In deriving the relations, it is assumed that individual effects of the component motions can be superimposed, which is tantamount to assuming that the downwash distributions due to rotation and translational acceleration may be superimposed.[5]

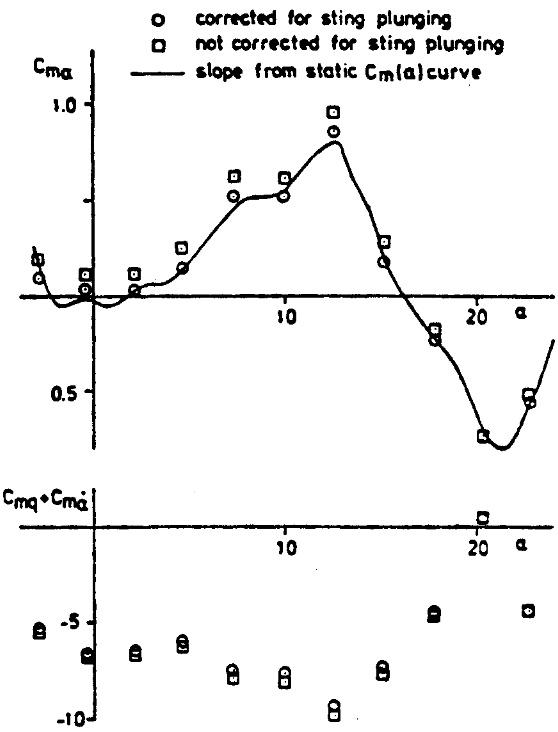

Fig. 4 Effect of sting plunging on $\bar{C}_{mq}$ and $C_{m\alpha}$ for the SDM at Mach 0.6.[9]

For small angles of incidence and low rates, this is probably justifiable, but when the flow becomes substantially separated due to high angles of attack and/or high rotation rates, the validity of these relations could break down. Nevertheless, such equations would still be considered useful in the present context, since the actual shift in the coordinate axes is usually very small. Moreover, the existing methods for correction of sting plunging effects [Eqs. (1-5)] are equally limited by reliance on the superposition principle.

Consider a shift of the reference axis perpendicular to the force vector and along the longitudinal axis of the model. Then the contributions due to axial force and its derivatives are naturally zero, while C_ℓ and its derivatives with respect to α, β, $\dot\alpha$, $\dot\beta$, and p are unaffected by a shift along x. The general transformation equations[5,7] may then be simplified to the following form:

$$C^*_{\ell i} = C'_{\ell i} \qquad i=\alpha,\beta,\dot\alpha,\dot\beta,p$$

$$C^*_{mi} = C'_{mi} + \frac{x}{\bar c} C_{Zi}$$

$$C^*_{ni} = C'_{ni} - \frac{x}{b} C_{Yi} \tag{16}$$

The remaining dynamic moment derivatives transform as follows:

$$C^*_{\ell q} = C'_{\ell q} + 2\frac{\cos\alpha}{\cos\beta}\frac{x}{b} C^*_{\ell\alpha}$$

$$+2\frac{x}{b}\sin\alpha(-C^*_{\ell\beta}\sin\beta + 2C^*_\ell\cos\beta)$$

$$C^*_{\ell r} = C'_{\ell r} - 2\frac{x}{b}(C^*_{\ell\beta}\cos\beta + 2C^*_\ell\sin\beta) \tag{17}$$

$$C^*_{mq} = C'_{mq} + \frac{x}{\bar c} C_{Zq} + 2\frac{\cos\alpha}{\cos\beta}\frac{x}{\bar c} C^*_{m\alpha}$$

$$+2\frac{x}{\bar c}\sin\alpha(-C^*_{m\beta}\sin\beta + 2C^*_m\cos\beta)$$

$$C^*_{mr} = C'_{mr} + \frac{x}{\bar c} C_{Zr} - 2\frac{x}{\bar c}(C^*_{m\beta}\cos\beta + 2C^*_m\sin\beta) \tag{18}$$

$$C^*_{nq} = C'_{nq} - \frac{x}{b} C_{Yq} + 2\frac{\cos\alpha}{\cos\beta}\frac{x}{b} C^*_{n\alpha}$$

$$+2\frac{x}{b}\sin\alpha(-C^*_{n\beta}\sin\beta + 2C^*_n\cos\beta)$$

$$C^*_{nr} = C'_{nr} - \frac{x}{b} C_{Yr} - 2\frac{x}{b}(C^*_{n\beta}\cos\beta + 2C^*_n\sin\beta) \tag{19}$$

The transformed static derivatives and moment coefficients appearing in these equations are defined in Eqs. (16). The fixed-axis dynamic derivatives are the sum of the rotational and translational contributions. For an arbitrary rotation,[8]

$$\dot\alpha = q - (p\cos\alpha + r\sin\alpha)\tan\beta$$

$$\dot\beta = p\sin\alpha - r\cos\alpha \tag{20}$$

This shows that both the $\dot\alpha$ and $\dot\beta$ terms are included in the derivatives due to oscillation in yaw. The composite fixed-axis derivatives are defined as

$$\bar C_{iq} = C_{iq} + C_{i\dot\alpha}$$

$$\bar C_{ir} = C_{ir} - C_{i\dot\beta}\cos\alpha - C_{i\dot\alpha}\left(\frac{\bar c}{b}\sin\alpha\tan\beta\right) \tag{21}$$

where $i=\ell,m,n$. Thus, combining the rotational and translational dynamic derivatives given by Eqs. (17-19) and (16), respectively, in Eqs. (21), a set of equations is obtained similar to Eqs. (17-19) but with the rotational dynamic derivatives replaced by their composite counterparts denoted by the superscript ($\bar{}$).

General Corrections for Support Oscillation

The foregoing analysis may now be extended to the general case of the influence at nonzero α and β of a translational DOF on the direct, cross, and cross-coupling derivatives. Referring to Fig. 3, the kinematic relationships between the moments measured at C and P [Eq. (7)] are

$$M' = M - Zx_c$$

$$N' = N + Yx_c \tag{22}$$

which yields the following set of derivatives referred to the rotation center:

$$\bar C'_{\ell q} = \bar C_{\ell q\dot z}; \qquad C'_{\ell\alpha} = C_{\ell\alpha z}$$

$$\bar C'_{\ell r} = \bar C_{\ell r\dot y}; \qquad C'_{\ell\beta} = C_{\ell\beta y} \tag{23}$$

$$\bar C'_{mq} = \bar C_{mq\dot z} - \bar C_{Zq}\frac{x_c}{\bar c}; \qquad C'_{m\alpha} = C_{m\alpha z} - C_{Z\alpha}\frac{x_c}{\bar c} \tag{24}$$

$$\bar C'_{mr} = \bar C_{mr\dot y} - \bar C_{Zr}\frac{x_c}{\bar c}; \qquad C'_{m\beta} = C_{m\beta y} - C_{Z\beta}\frac{x_c}{\bar c} \tag{25}$$

$$\bar C'_{nq} = \bar C_{nq\dot z} + \bar C_{Yq}\frac{x_c}{b}; \qquad C'_{n\alpha} = C_{n\alpha z} + C_{Y\alpha}\frac{x_c}{b} \tag{26}$$

$$\bar C'_{nr} = \bar C_{nr\dot y} + \bar C_{Yr}\frac{x_c}{b}; \qquad C'_{n\beta} = C_{n\beta y} + C_{Y\beta}\frac{x_c}{b} \tag{27}$$

Replacing the dynamic derivatives in the transformation equations [Eqs. (16-19)] with their composite fixed-axis counterparts, the derivatives given in Eqs. (23-27) may be transformed to the reference center. Then, introducing $\xi_c = |x_c|/\bar c$ as before and rearranging, the following equations are obtained relating the values of the corrected derivatives to their counterparts measured in the presence of support motion:

$$C_{\ell iz} = C_{\ell iy} = C_{\ell i}; \qquad i=\alpha,\beta,\dot\alpha,\dot\beta,\mathrm{p}$$

$$C_{miz} = C_{mi}$$

$$C_{niy} = C_{ni} \tag{28}$$

$$\bar C_{\ell q\dot z} = \bar C_{\ell q} \mp 2\frac{\bar c}{b}\xi_c\frac{\cos\alpha}{\cos\beta} C_{\ell\alpha}$$

$$\pm 2\frac{\bar c}{b}\xi_c\sin\alpha(C_{\ell\beta}\sin\beta - 2C_\ell\cos\beta)$$

$$\bar C_{\ell r\dot y} = \bar C_{\ell r} \pm 2\frac{\bar c}{b}\xi_c(C_{\ell\beta}\cos\beta + 2C_\ell\sin\beta) \tag{29}$$

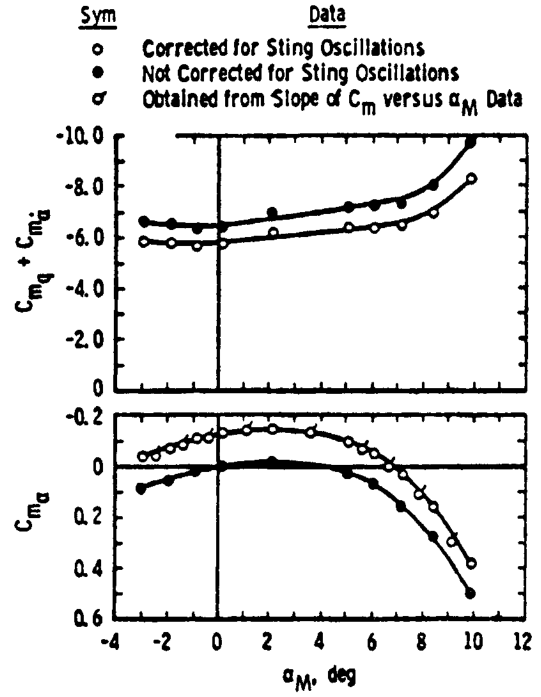

Fig. 5 Effect of sting plunging for AGARD Model C at Mach 2.5.[2]

$$\bar{C}_{mq\dot{z}} = \bar{C}_{mq} \mp 2\xi_c \frac{\cos\alpha}{\cos\beta} C_{m\alpha}$$

$$\pm 2\xi_c \sin\alpha (C_{m\beta} \sin\beta - 2C_m \cos\beta)$$

$$\bar{C}_{mr\dot{y}} = \bar{C}_{mr} \pm 2\xi_c (C_{m\beta} \cos\beta + 2C_m \sin\beta) \tag{30}$$

$$\bar{C}_{nq\dot{z}} = \bar{C}_{nq} \mp 2 \frac{\bar{c}}{b} \xi_c \frac{\cos\alpha}{\cos\beta} C_{n\alpha}$$

$$\pm 2 \frac{\bar{c}}{b} \xi_c \sin\alpha (C_{n\beta} \sin\beta - 2C_n \cos\beta)$$

$$\bar{C}_{nr\dot{y}} = \bar{C}_{nr} \pm 2 \frac{\bar{c}}{b} \xi_c (C_{n\beta} \cos\beta + 2C_n \sin\beta) \tag{31}$$

where $\left\{ \begin{matrix} \bar{\omega}_{z0} > \bar{\omega} \\ \bar{\omega}_{z0} < \bar{\omega} \end{matrix} \right\}$ in Eqs. (28–31).

Equations (28–31) constitute a complete set of corrections for sting oscillation in planar, free-, or forced-oscillation tests.

Discussion

At low angles of attack, the $\sin\alpha$ terms in Eqs. (29–31) are small, so that the effects of sting oscillation on the dynamic derivatives are given in each case by a term dependent on ξ_c and the corresponding static derivatives. Equation (30) shows that, for a statically stable model, the plunging DOF will decrease the damping for a supercritical sting and increase it in the subcritical case. As already noted, the absence of any effect on the pitching-moment slope [Eq. (28)] is the consequence of neglecting the frequency in the transformation equations. In most cases, this contribution is very small, and larger effects on the static stability, such as those reported in Refs. 2 and 9, are more likely to have been produced by an unbalanced mass [Eq. (5)].

The effect on the damping agrees with the earlier observations of Ericsson[1] and Burt and Uselton.[2] An increase in damping due to sting plunging was measured by Jansson and Torngren[9] on the Standard Dynamics Model (SDM) (see Fig. 4). However, their interpretation that the sting plunging correction agrees with the results of Burt and Uselton[2] (Fig. 5) is misleading, as the FFA measurements were obtained on a supercritical sting ($\bar{\omega}_{z0} \simeq 2\bar{\omega}$) and the Arnold Engineering Development Center data on a subcritical sting. The agreement was due to the fact that the pitching-moment slopes had different signs in the two tests. In the National Academy of Engineering SDM experiment,[8] the positive-axis shift measured in the pitch-oscillation mode (Fig. 6) also confirmed the predicted increased damping, but the effect was very small, as might have been expected for a supercritical sting arrangement having $\bar{\omega}_{z0} > 3\bar{\omega}$.

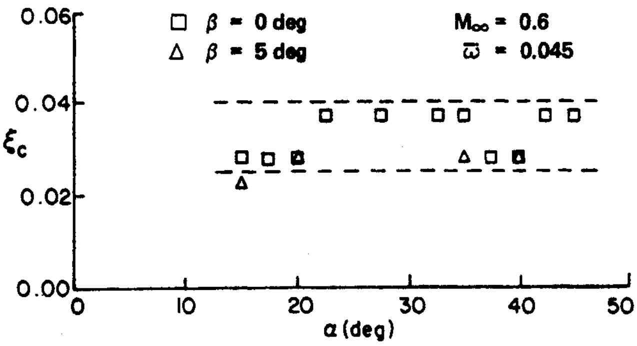

Fig. 6 Effective rotation center in SDM pitch-oscillation tests.[8]

Looking now at the more complete equations, it is noted first that the effect on the yaw damping derivative [Eq. (31)] is, in a qualitative sense, the same as for pitch oscillation, namely that the lateral oscillation decreases the damping for a configuration statically stable in yaw (when $\bar{\omega}_{z0} > \bar{\omega}$), at least at low α. The effects on the cross-coupling derivatives $\bar{C}_{\ell q}$ and $\bar{C}_{nq}$ [Eqs. (29) and (31)] are similar to that for $\bar{C}_{mq}$, while the cross and cross-coupling derivatives $\bar{C}_{\ell r}$ and $\bar{C}_{mr}$ behave similarly to $\bar{C}_{nr}$.

When the dynamic characteristics are suspected of being rate-dependent, the frequency effects will have to be adequately modeled in addition to effects of incidence. While the veracity of the corrections obtained by this approach is established by the completeness of the transformation model, the basic premise underlying the method, namely, that the true fixed-axis derivatives are determined at the axis of rotation, is clearly always valid. Therefore, if the derivatives can be referred to the shifted axis [Eqs. (23–27)], no transformation is required.

Practical Considerations

The indications obtained that the effect of the axis shift on the measured direct derivatives of the SDM are very small substantiate the analysis of Ref. 4, in which it is concluded that the use of a supercritical sting can obviate the need for sting plunging corrections in many cases; the undamping contribution due to a supercritical sting is always smaller than its subcritical counterpart when $\bar{\omega}_{z0} > \sqrt{2}\bar{\omega}$. This was a desirable expedient when there was a need to determine cross and cross-coupling derivatives. With the advent of a complete set of support-oscillation correction equations, the use of a supercritical sting is, however, no longer necessary.

Under conditions of high angles of attack and/or high rotation rates, various effects might be present which could rule out the axis transformation, including: 1) significant frequency effects, 2) aerodynamic effects due to motion coupling associated with sting oscillation, 3) dynamic sting interference produced by the relative motion of the model and support, and 4) flow unsteadiness that causes the sting displacements to be erratic. Under conditions where any of these effects are severe, analytical correction for sting oscillation is very difficult, and it becomes necessary to account for the effects in the experimental procedure.

One possibility would be to determine the effect of reference axis location $\hat{x}$ experimentally rather than analytically, a procedure that would be advisable in any event when significant nonlinear effects are suspected. The corrected data would then be obtained through interpolation at $\hat{x} = \hat{x}_c$.

A second approach would be to perform additional tests to determine the axis shift as a function of the model incidence angles α, β and then to adjust the model position on the balance by means of ring spacers to compensate for the expected axis shift at each test condition. Since the plunging contribution vanishes at the rotation center, the sting oscillation effects will be eliminated when the reference and rotation axes are coincident. However, this procedure involves a substantial addition to a test program including the manufacture of additional test hardware, and should be followed only when the analytical corrections are ruled out.

Conclusions

1) The corrections for sting plunging effects were shown to be equivalent to an aerodynamic axis transformation from the inertial rotation center.

2) The results are in agreement with corrections derived from the linear equations of motion at the same level of simplification.

3) A complete set of equations expressing the corrected stability derivatives in terms of their counterparts measured in the presence of support oscillation was derived, subject only to the restrictions of small angles, noncritical sting frequency, and low structural damping.

4) The method is quite general, being applicable to free- or forced-oscillation tests in any angular degree of freedom, and may be extended to nonlinear parameters (rather than derivatives), provided that appropriate transfer relations can be derived.

5) With the availability of the set of correction equations, more slender, subcritical sting configurations may be used.

6) In situations where analytical corrections are ruled out, support oscillation effects can be eliminated by compensating for the axis shift mechanically rather than analytically.

References

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